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## COMPLEX SIGNALS PARAMETERS OPTIMIZATION ON THE BASE OF LINEAR APPROXIMATIONS USING THE GRADIENT METHOD AND NEWTON'S METHOD

*The article examines the effectiveness of the gradient descent and Newton methods for optimizing the parameters of ensembles of complex signals. Algorithms have been developed and implemented that increase the accuracy of setting parameters and ensure reasonable optimization of spectral, temporal and statistical characteristics of signals. The effectiveness of the application of the methods was confirmed experimentally on the example of reducing the error and increasing the level of immunity. The obtained results substantiate the improvement of the parameters of complex signals, which proves the efficiency of use for wireless telecommunication systems in order to ensure stable and reliable operation in conditions of dynamic changes in the environment and a high level of interference.*

*The article compares the mathematical methods of optimization, namely the gradient method and Newton's method, proposes mathematical models and constructs algorithms that empirically prove the effectiveness of the application of the studied mathematical methods in the specified scientific area - for optimizing the parameters of ensembles of complex signals. Scientific works [1-6, 9, 12] present algorithms based on the gradient method and Newton's method, but they do not consider in detail the comparative analysis of the effectiveness of these methods for optimizing the parameters of ensembles of complex signals for implementation in various scientific and practical tasks. The effectiveness of the algorithms proposed in the article was confirmed experimentally, which made it possible to reduce the error and improve the characteristics of ensembles of complex signals.*

*As a result of the experiments using the methods of gradient descent and Newton, a significant reduction of the error and an improvement of the stability of the signals were achieved. Newton's method reduced the error from 0.1 to 0.0027, justifying the high accuracy of setting the signal parameters. The gradient descent method provided a stable reduction of the gradient norm from 12.75 to less than 1.23, effectively reducing the interference level, i.e. increasing the interference immunity.*

**Keywords:** linear approximations, ensembles of complex signals, gradient method, Newton's method, gradient norm, objective function, optimization of signal parameters, iterative algorithm, noise immunity.

### INTRODUCTION

Researching gradient descent and Newton's methods is relevant for optimizing the parameters of complex signal ensembles because these methods provide high accuracy and efficiency in tuning spectral, temporal, and statistical characteristics of signals [1-13].

The application of these methods can significantly enhance the noise immunity of complex signal ensembles, which is critical for the functioning of wireless telecommunications systems. In the face of constant environmental changes and the presence of various interferences and obstacles,

these systems require reliable and stable operation, which can be achieved by optimizing signal parameters using adaptive algorithms such as gradient descent and Newton's method.

Solving the problem of optimizing the parameters of complex signal ensembles is essential for ensuring the efficiency and reliability of modern wireless communication systems.

**The object of study** is the process of optimizing the parameters of complex signal ensembles with given parameters.

**The subjects of study** are the algorithms and methods of optimization, particularly gradient descent and Newton's methods.

**The purpose of this work** is to evaluate the effectiveness of gradient descent and Newton's methods, as well as their comparative analysis with the Nelder-Mead method for optimizing the parameters of complex signal ensembles.

## 1 PROBLEM STATEMENT

In practice, alongside the Nelder-Mead method, which is based on a direct search algorithm and the use of a simplex to determine the main direction towards the minimum point, the gradient method is also used in scientific research. This method is an iterative search method based on the use of the gradient of the objective function to determine the optimal direction of movement. The gradient method can be effectively used to optimize a wide range of parameters of complex signal ensembles, in particular, the signal's spectral density to meet certain specified criteria such as [1, 3, 5]:

- ensemble properties of signals, namely: mean value, dispersion, and autocorrelation parameters;
- signal noise immunity properties, specifically: uniform energy distribution across frequencies or resistance to various types of interference;
- spectral characteristics of signals, such as the presence or absence of certain frequency components.

Additionally, the gradient method can be applied to optimize the temporal characteristics of complex ensemble formations:

- optimization of parameters such as time shift and pulse length, allowing control over the temporal structure of complex signals and facilitating synchronization and other aspects of data transmission;
- application of various «window functions», enabling the modification of signal waveforms, thereby reducing visual artifacts or other unwanted effects that may arise during operation and impact signal quality;
- optimization of signal filters to effectively remove frequencies that cause interference and disruptions or create artifacts, thereby significantly improving the overall quality and performance of the system.

Additional examples of using the gradient method for optimizing the parameters of complex signal ensembles include:

- optimization of modulation parameters: allows improving information transmission by adjusting the amplitude, frequency, or phase characteristics of signals to achieve better accuracy and resistance to interference;
- enhancing the energy efficiency of signals: by optimizing the energy characteristics of signals, the gradient method can help reduce the system's power consumption, which is critically important for the operation of wireless systems;
- improvement of signal synchronization: ensuring precise alignment of signals in time, which is essential for coherent signal processing and avoiding timing errors in communication systems;
- adaptive beamforming in antenna arrays: optimizing the directionality and strength of signal transmission to improve reception quality and reduce the impact of interference from unwanted directions.

The preliminary stage of the gradient method algorithm is the determination of the objective function, which takes place before the start of the iterative optimization process. For this, it is necessary to define an objective function  $f(x)$  that, for example, evaluates the distance between the parameter vectors of spectral density, the target density  $x_{target}$ , and the robustness density  $x_{robust}$ . Mathematically, the formula will be as follows [1, 2]:

$$f(x) = \|x - x_{target}\|^2 + \|x - x_{robust}\|^2 \quad (1)$$

where  $\| \cdot \|$  – Euclidean norm;

$x, x_{target}, x_{robust}$  – spectral density vectors.

Based on this, the gradient of the objective function will have the form [4,5]:

$$\nabla f(x) = \begin{pmatrix} \frac{\partial f(x)}{\partial x_1} \\ \dots \\ \frac{\partial f(x_m)}{\partial x_m} \end{pmatrix} = \begin{pmatrix} -2(x_{target} - x) \\ \dots \\ -2(x_{robust} - x) \end{pmatrix} \quad (2)$$

## 2 REVIEW OF THE LITERATURE

The works [1-13] analyzed within this study are devoted to the theoretical foundations and practical applications of various aspects of optimizing the parameters of complex signals using linear approximations with gradient descent and Newton's methods. Significant results have been achieved in all these studies, which enhance the understanding of the practical efficiency of different optimization methods. The studies [1, 4-6] found that the gradient method is effective for optimizing the spectral characteristics of signals, as well as for solving problems of pattern

recognition and source localization. For example, it has been established that this method reduces errors in pattern recognition and improves localization accuracy.

In the studies [2, 3, 5, 6, 13], the practical application of the Newton method justified that this method provides faster convergence compared to gradient descent, especially when optimizing the parameters of signal ensembles. It was found that the Newton method can significantly reduce the number of iterations needed to achieve an optimal solution.

The studies [7, 8] empirically confirmed the effectiveness of optimizing filter parameters using the Nelder-Mead and Levenberg-Marquardt algorithms. The differential evolution algorithm considered in [8] also showed high efficiency in optimizing synthesized signals, which in practice ensures high accuracy and stability of scientific results.

In the studies [9, 10, 11], specific methods and approaches for optimizing signals and processes were investigated. For example, [9] found that the process of signal separation based on a first-order linear complex autoregressive process is effective for improving signal quality. In [10], the feasibility of using polynomials with hybrid gradient descent and Newton's method was confirmed, which improves the convergence and accuracy of optimization. The study [11] showed that using Bregman distances for regularizing the gradient of the Newton method enhances the stability of the algorithm.

Through the analysis in study [12], the optimal total throughput for SWIPT NOMA systems was identified. This can be used to improve the efficiency of wireless telecommunication systems.

The use of the proposed methods will improve the signal characteristics and ensure a high level of noise immunity and performance in complex dynamic telecommunication systems that require high reliability and adaptation to changing environmental conditions.

The general algorithm of the gradient method is presented in Fig. 1 (without the stage of determining the objective function [7, 9]).

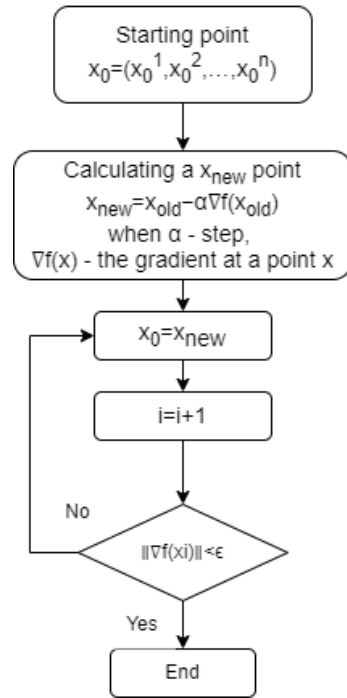


Fig. 1 – Optimization algorithm based on gradients

The generalized approach to adjusting the parameters of the gradient-based method for optimizing the signal's spectral density is presented in Table 1.

Table 1  
An example of the algorithm for changing parameters based on gradients

**3 MATERIALS AND METHODS**

Iteration	Calculation	Parameter Values
0	$x_0$	$x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$
1	$x_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - \eta \nabla f(x_0) = \begin{pmatrix} 0,5 \\ 0,5 \end{pmatrix}$	$x_1 = \begin{pmatrix} 0,5 \\ 0,5 \end{pmatrix}$
2	$x_2 = \begin{pmatrix} 0,5 \\ 0,5 \end{pmatrix} - \eta \nabla f(x_1) = \begin{pmatrix} 0,25 \\ 0,25 \end{pmatrix}$	$x_2 = \begin{pmatrix} 0,25 \\ 0,25 \end{pmatrix}$
....	....	....
$n$	$x_n = \begin{pmatrix} x_n^1 \\ x_n^2 \end{pmatrix} = \begin{pmatrix} x_{n-1}^1 - \eta \nabla f(x_{n-1}) \\ x_{n-1}^2 - \eta \nabla f(x_{n-1}) \end{pmatrix}$	$x_n = \begin{pmatrix} x_n^1 \\ x_n^2 \end{pmatrix}$

Starting with an initial value  $x_0 = (1,1)$ , we then determine the gradient of the objective function. In the next step, we calculate a new point  $x_{new}$ , with the condition that each such new point becomes the current value for the next iteration, and the iteration counter increases at each step of the algorithm. It is mandatory to check the stopping condition of the algorithm during the computation, specifically: if the norm of the gradient is

less than or equal to the given threshold  $\epsilon$ , the algorithm stops.

**4 EXPERIMENTS**

Fig. 2 and Fig. 3 show an example of applying gradient-based optimization to obtain improved parameters of the signal's spectral density.

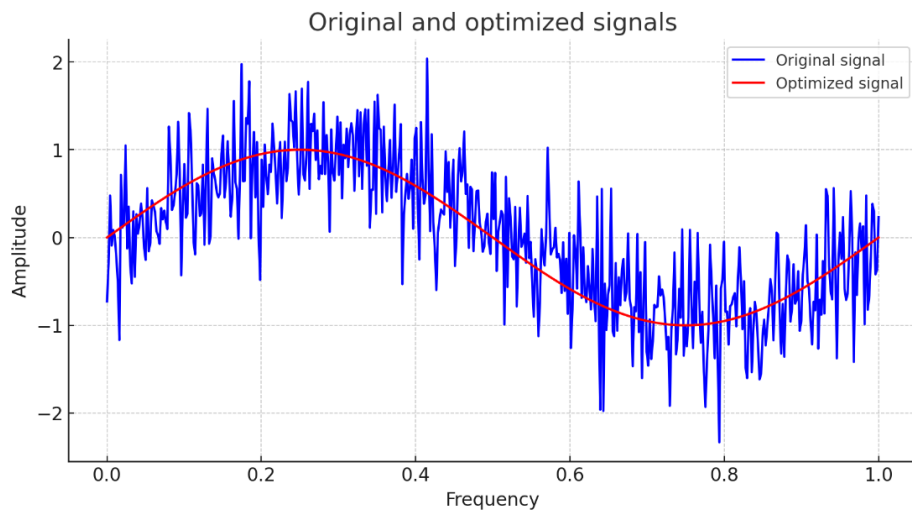


Fig. 2 – Analysis of signal changes after optimization

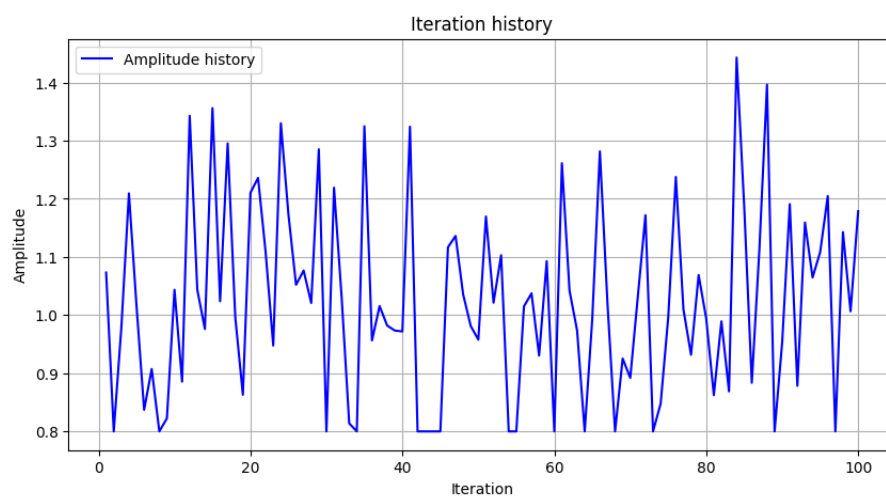


Fig. 3 – Dynamics of signal amplitude changes during iterations

Table 2 shows how the signal parameters and the gradient norm changed with each iteration. The gradient norm is a characteristic that indicates the change in the function of the gradient vector length at a specific point. In our case, the gradient norm considers how close the signal parameter optimization algorithm is to the given minimum point  $\epsilon$  [9, 11].

Table 2  
Dynamics of signal parameters on gradient norms by iterations

Iteration	Signal parameters on current iteration (first 5 values, total 500 points)	Gradient norm
0	1,13, 0,85, 1,05, 1,13, 1,38....	12,75
1	1,02, 0,76, 0,94, 1,02, 1,25	11,50
2	0,92, 0,68, 0,84,	10,35

	0,92, 1,13	
3	0,83, 0,61, 0,75, 0,83, 1,02	9,30
4	0,75, 0,55, 0,67, 0,75, 0,92	8,35
.....	.....	.....
n	0,10, 0,01, 0,20, 0,15, 0,01	< 1.23

Table 3  
Optimization of signal parameters by the gradient method

Iteration	1	2	3	4	5	6	7	8	9	10	...	100
Amplitude	1,2	0,9	1,1	0,8	1,0	1,3	0,9	1,1	1,4	1,2	...	1,0

The optimization process using the gradient method can effectively adjust the signal amplitude, improving the level of noise immunity. Changes in amplitude at each

iteration indicate the process of approaching the optimal level, which helps achieve better characteristics and stability under various interference conditions. Such results are achieved by using the gradient of the objective function, which indicates the direction and magnitude of the necessary changes to improve the signal ensemble. The gradient shows how quickly and in which direction the signal ensemble parameters need to be changed to minimize the deviation from the target function value. As a result, the amplitude of the signal ensemble is gradually adjusted (improved) during the iterations, approaching the optimal value, which overall contributes to ensuring a high level of noise immunity [11].

Let's consider a situation where the objective function  $f(x)$  evaluates the distance between the parameter vectors of the spectral density  $x$  and the target density  $x_{target} = \begin{matrix} 0,7 \\ 0,7 \end{matrix}$  and the robustness density  $x_{robust} = \begin{matrix} 0,2 \\ 0,2 \end{matrix}$ . Then, the gradient of this function will be as follows:

$$\nabla f(x) = \begin{pmatrix} \frac{\partial f(x_1)}{\partial f(x_2)} \end{pmatrix} = \begin{pmatrix} -2(x^1_{target} - x^1) \\ -2(x^2_{robust} - x^2) \end{pmatrix} \quad (3)$$

Let's determine the condition for the experiment that the gradient descent method optimizes with a step  $\eta = 0,1$ , then the change in parameters (for example, signal amplitude and noise level) will take the form (Table 4).

At each step of the iteration, for example, the signal amplitude (parameter 1) will change according to the formula (as well as other parameters) [12]:

$$A_{new} = A_{old} - \eta \nabla f(A_{old}) \quad (4)$$

where  $\eta$  – the step coefficient, the learning rate, determines the change of the parameter at each step.

Table 4  
Dynamics of changes in signal parameters during optimization

Iteration	Parameter 1 (signal amplitude)	Parameter 2 (noise level)
0	1	1
1	0,5	0,6
2	0,25	0,35
3	0,125	0,2
4	0,0625	0,1
5	0,03125	0,05

As calculations show, under this condition, parameter 1 is halved at each iteration, which indicates a stable optimization process aimed at reducing the amplitude to achieve signal stability. And parameter 2, decreases at different stages, is not so stable, which indicates a more complex optimization process, where the noise level decreases depending on other factors, such as

frequency components or other parameters of the signal (Fig. 4).

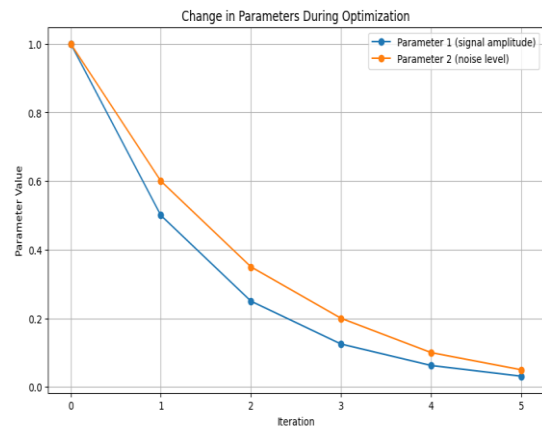


Fig. 4 – Iterative optimization of signal parameters (gradient method)

The original signal is a sine wave with a frequency between 0 and 1. The optimized signal can be obtained by applying gradient descent to the original signal. The algorithm of the gradient descent method is built in such a way that it «tries to find» such a signal amplitude that will minimize the difference between the original signal and noise (reduce the effect of noise on the signal). The amplitude of the signal changes significantly in the first few iterations (from 1 to 0.5). As the iterations continue, the signal amplitude stabilizes (0.0625), which indicates the effectiveness of the optimization method.

The effectiveness of the gradient descent method depends on the size of the gradient descent step  $\eta$ . If the step  $\eta$  is too small, the method may be inefficient, as it will require many iterations to achieve the desired result. If the step  $\eta$  is too large, the method may be unstable because it may «jump» through the minimum point of the objective function.

In this experimental case, at step  $\eta = 0,1$ , it took only 5 iterations to achieve the desired result, and the method improved the parameters by 10% at each iteration.

The gradient descent method is an effective way to optimize the spectral density of a signal, as it allows achieving the desired result in a small number of iterations. As a result of applying the method, the synthesized signals acquired appropriate ensemble properties and noise immunity characteristics, with the amplitude values of the signals increasing, which provided a higher level of noise immunity. The calculations show that choosing the optimal step size for gradient descent allows efficiently achieving the necessary experimental results.

Another effective method of optimization with constraints, using approximation by linear functions, is Newton's method. It can also be used for complex ensembles of synthesized signals. This method is effective for objective functions with a non-smooth graph, which

are often encountered in practice in signal synthesis [10, 11, 13]. Newton's method uses a quadratic approximation of the objective function  $f(x)$  at the current evaluation point  $x_k$  [13]:

$$Q(x) = f(x_k) + \nabla f(x_k)^T(x - x_k) + \frac{1}{2}(x - x_k)^T H(x_k)(x - x_k) \tag{5}$$

where  $f(x_k)$  – the value of the objective function at a point  $x_k$ ;

$\nabla f(x_k)$  – the gradient of the objective function at a point  $x_k$ ;

$H(x_k)$  – hessian – the matrix of the second partial derivatives of the function at the point  $x_k$

Quadratic approximation allows quickly finding the directions of the steepest descent of the objective function, making the Newton method effective for fast convergence in regions where quadratic approximation is truly effective [11]. By using second-order derivatives in the calculations, the Newton method provides a more accurate approximation to the minimum of the objective function compared to methods that use first-order derivatives.

Optimization using the Newton method is presented in Fig. 5 and Table 5.

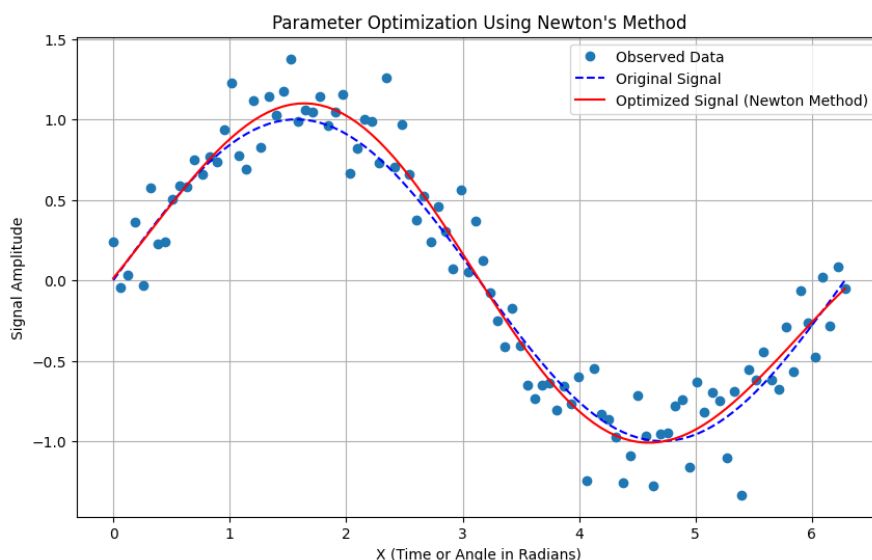


Fig. 5 – Optimization of signal parameters according to Newton's method

In Fig. 5 shows the optimization of the signal parameters by Newton's method, namely, reducing the error and improving the fit to the observed data. The observed data (black dots) are approximated by the original sinusoidal signal (blue dashed line) and the optimized signal (red solid line). Newton's method is an effective tool for optimizing the parameters of signal ensembles, in particular the frequency, in order to achieve the maximum fit to the data. The process begins with an initial assumption about the frequency, which in this example is set to 0.5. According to Newton's method, the derivative of the frequency is used to find a new value that is more optimal and better corresponds to the given data.

At each iteration step, Newton's method calculates a new frequency value, reducing the error. After the first iteration, the new frequency value is 0.45, with an error of 0.075. The process of iterations continues until the accuracy specified by the experiment is reached. In this example, optimization by Newton's method is achieved in 10 iterations, reducing the error from 0.1 to 0.0027. The results of the calculations are presented in the Table. 5.

Table 5. Results of calculations by Newton's method

Iteration	Frequency	The calculation error
0	0,5	0,1
1	0,45	0,075
2	0,475	0,05625
3	0,46875	0,0421875
4	0,46484375	0,03125
5	0,462953125	0,02265625
6	0,4619140625	0,015625
7	0,46123046875	0,0107421875
8	0,4608154296875	0,0068359375
9	0,4605712890625	0,004296875



1	0,460418701	0,002685
0	171875	546875

Thus, the use of the Newton method allows for a significant reduction in error from 10% to 0.027%, which corresponds to an overall error reduction of 99%. This demonstrates the high efficiency of the Newton method for optimizing the parameters of complex signal ensembles. The high accuracy of this method is ensured by the use of second-order derivatives, which provide more precise approximations to the minimum of the objective function compared to methods that use first-order derivatives.

Additionally, the Newton method shows high performance in cases where the objective function is smooth. This makes the Newton method particularly useful for tasks requiring fast and accurate optimization of parameters, such as tuning the frequency, phase, and amplitude of signals in complex and dynamic radio environments.

Newton's method is an effective way to optimize the parameters of ensembles of complex signals in problems with high accuracy. However, this method may be less efficient for the objective function with a smooth graph, as it requires an increase in the number of iterations to achieve the optimal result. To solve such practical problems, it is necessary to consider the use of alternative

optimization methods, such as the gradient method or the Nelder-Mead method.

However, Newton's method has a number of disadvantages. It is sensitive to the choice of the initial value of  $x_0$ , which can affect the optimization results. If the initial value is far from optimal, the method may converge slowly or not at all. Therefore, when using this method, it is important to choose an initial value (solution) that is close to the optimal one. This is not always appropriate. In addition, Newton's method can be slow for high-dimensional objective functions due to the need to calculate and invert the Hessian, which is computationally challenging. In cases where there is a need to take into account the above parameters, it is more reasonable to use methods based on gradients or Nelder-Mead.

It should also be noted that Newton's method may be less efficient in the case of functions with multiple local minima, where it may stop at a local minimum instead of a global one. This limits its application in problems with high complexity of the topography of the objective function.

A comparison of the effectiveness of the Nelder-Mead, gradient descent, and Newton methods for optimizing complex signal ensembles, highlighting the main advantages and disadvantages of each method, is presented in the table. 6.

Table 6

Comparison of the effectiveness of the gradient method and the Nelder-Mead method

Properties	Method of Nelder - Mead	Method based on gradients	Newton's method
Strengths	<ol style="list-style-type: none"> <li>Does not require the calculation of the gradient of the objective function</li> <li>Has high resistance to noise and measurement errors.</li> <li>Works well when there is limited information about the target function.</li> </ol>	<ol style="list-style-type: none"> <li>Effective for objective functions with a non-smooth graph</li> <li>Has the ability to build a fast algorithm for high-dimensional objective functions.</li> <li>Flexibility in adjusting the learning speed, which helps to avoid «falling into» local minima and ensures faster achievement of the general («global») minimum.</li> </ol>	<ol style="list-style-type: none"> <li>It has the ability to quickly achieve accurate parameters</li> <li>Effective for objective functions with a non-smooth graph</li> <li>When calculating, derivatives of the second order are used for accuracy</li> </ol>
Weaknesses	<ol style="list-style-type: none"> <li>May be ineffective for objective functions with a non-smooth graph.</li> <li>Can be slow for high-dimensional objective functions.</li> <li>May get «stuck» in local minima for complex functions.</li> </ol>	<ol style="list-style-type: none"> <li>Requires the calculation of the gradient of the objective function, which necessitates complex computation algorithms.</li> <li>Is sensitive to the choice of the initial point <math>x_0</math>, which can affect optimization results.</li> <li>Requires the adjustment of the step size <math>\eta</math>, which can impact</li> </ol>	<ol style="list-style-type: none"> <li>The method requires calculation and inversion of the Hessian</li> <li>Is sensitive to the choice of initial values</li> </ol>

		the stability and speed of convergence.	
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The Nelder-Mead method is more reasonable for use in cases where the objective function has a smooth graph and noise and disturbances become a significant problem (which is usually the case in practice). This method does not require the calculation of the gradient of the objective function, which is a significant advantage, because the calculation of the gradient is difficult or expensive. In addition, the Nelder-Mead method is based on direct search and use of the simplex, which provides good resistance to interference and can work effectively under conditions of limited information about the objective function. This makes it optimal for solving problems where the objective function may have multiple local minima or be difficult to analyze due to various interferences and obstacles.

Gradient methods can be effective when the graph of the objective function is not smooth, and disturbances do not significantly affect the process. Gradient methods, such as gradient descent, can quickly converge to the optimal solution, especially for high-dimensional objective functions. An important condition for the effectiveness of gradient methods is the condition for calculating the gradient. That is, it is always necessary to consider the situation to what extent it is costly or not costly. The use of gradient methods is the optimal solution for problems with a large number of parameters, since they can be adapted to quickly converge to the minimum of the objective function.

## 5 RESULTS

As shown in fig. 1-3 and in table. 2-3, the gradient method proves its effectiveness in adjusting the signal amplitude and noise level, namely according to the characteristics:

1. Increasing signal similarity. Fig. 2 shows that the initial amplitude of the signal varies significantly during the first few iterations and then stabilizes at a value of 0.0625. This indicates the effectiveness of the gradient descent method in reducing the influence of interference on the signal.

2. Reduction of signal amplitude. In the table 4 shows that the signal amplitude is halved at each iteration, demonstrating a stable optimization process aimed at achieving signal stability.

3. Reduction of noise level. Parameter 2 (noise level) decreases at different stages of the optimization, indicating a more complex optimization process where the noise level is reduced depending on other factors, such as frequency components or other signal parameters (Fig. 3).

As shown in fig. 4-5 and in table. 6, Newton's method also proves its high efficiency in the tasks of optimizing signal parameters. Namely, in fig. 5, it can be seen that Newton's method significantly reduces the error and improves the fit to the observed data. The error

decreases from 0.1 to 0.0027 in 10 iterations, which corresponds to a 99% error reduction.

In the table 6 shows how the frequency of the signal changes at each iteration step, reducing the error. After the first iteration, the new frequency value is 0.45, and the iteration process continues until the specified accuracy is reached.

Due to the use of second-order derivatives, Newton's method provides a more accurate approximation to the minimum of the objective function compared to methods that use first-order derivatives.

The further development of this scientific research is the integration of optimization methods with innovative technologies of machine learning and artificial intelligence. The use of machine learning algorithms for preliminary data analysis and selection of optimal initial («starting») parameters for optimization algorithms, such as gradient descent and Newton's method, can significantly reduce the time to reach optimal results and increase their accuracy.

Another promising direction of research is the development of hybrid optimization methods, which combine the advantages of gradient methods and evolutionary search methods (Nelder-Mead method). This approach will make it possible to develop algorithms capable of effectively solving problems with a large number of parameters and different types of objective functions. The use of hybrid methods can significantly increase the efficiency of telecommunication systems, radar systems and other industries where high accuracy and reliability of operation are critical.

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## ЗАСТОСУВАННЯ ЛІНІЙНИХ АПРОКСИМАЦІЙ ДЛЯ ОПТИМІЗАЦІЇ ПАРАМЕТРІВ АНСАМБЛІВ СКЛАДНИХ СИГНАЛІВ ЗА ГРАДІЄНТНИМ МЕТОДОМ І МЕТОДОМ НЬЮТОНА

В статті досліджено ефективність методів градієнтного спуску та Ньютон для оптимізації параметрів ансамблів складних сигналів. Розроблено та впроваджено алгоритми, що підвищують точність налаштування параметрів та забезпечують обґрунтовану оптимізацію спектральних, часових і статистичних характеристик сигналів. Ефективність застосування методів підтверджено експериментальним шляхом на прикладі зменшення похибки та підвищення рівня завадостійкості. Отримані результати обґрунтовують удосконалення параметрів складних сигналів, що доводить ефективність використання для безпроводових телекомунікаційних систем з метою забезпечення стабільної та надійної роботи в умовах динамічних змін середовища та високого рівня завад.

У статті проведено порівняння математичних методів оптимізації, а саме градієнтного методу і методу Ньютон, запропоновано математичні моделі і побудовано алгоритми, які емпірично доводять ефективність застосування досліджених математичних методів в визначеній науковій області – для оптимізації параметрів ансамблів складних сигналів. У наукових роботах [1-6, 9, 12] представлено алгоритми, за градієнтним методом та методом Ньютон, але в них не розглянуто детально порівняльний аналіз ефективності цих методів для оптимізації параметрів ансамблів складних сигналів для реалізації в різних наукових і практичних задачах. Ефективність запропонованих в статті алгоритмів підтверджено експериментальним шляхом, що дозволило досягнути зменшення похибки та покращення характеристик ансамблів складних сигналів.

В результаті експериментів з використанням методів градієнтного спуску та Ньютон було досягнуто значного зменшення похибки та покращення стабільності сигналів. Метод Ньютон знизив похибку з 0,1 до 0,0027, обґрунтовуючи високу точність налаштування параметрів сигналу. Метод градієнтного спуску забезпечив стабільне зниження норми градієнта з 12,75 до менш ніж 1,23, ефективно зменшуючи рівень завад, тобто підвищуючи завадостійкість.

**Ключові слова:** лінійні апроксимації, ансамблі складних сигналів, градієнтний метод, метод Ньютон, норма градієнту, функція цілі, оптимізація параметрів сигналів, ітераційний алгоритм, завадостійкість.

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