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Electrical impedance of traction rails at audio frequency range

Based on the comparative analysis of the results of calculations of the series impedance of R65 type traction rails with 1520 mm gauge in the audiofrequency range according to Carson's method and complex depth of earth return method and literature data it have been confirmed applicability of the using of these methods for calculations of rail's impedance for 1520 mm gauge in audiofrequency range with using of correction factors.
Key words: *traction rails, frequency-dependent impedance, Carson's method, complex depth of earth return.*

Introduction

Knowledge of the accurate values of traction rails impedance and admittance over a wide frequency range is necessary for design and modeling audiofrequency track circuits [1], investigation distribution of traction return current in high-speed railway [2], induction coupling between jointless track circuits and track-circuit-reader antenna [3] etc. Results of the series rail impedance investigations were published in several papers notably in the works of Hill et al [4, 5]. Mariscotti et al [6, 7], and others. A theoretical analysis of the rail impedance frequency dependence is based mainly on the so-called Carson/Pollaczek model, or simply Carson's method proposed almost simultaneously by J.R. Carson [8] and F. Pollaczek [9] for determining the AC transmission line impedance considering earth return. They obtained equations for impedance under such specific assumptions:

- conductors are parallel and infinite length;
- return ground under wires is homogeneous and of constant resistivity;
- dielectric and magnetic permittivity of the ground are considered to be equal to unity;
- the displacement currents in the air and the ground are neglected.

Solutions of the equations in [8, 9] were obtained for quasi static transverse electromagnetic modes (TEM) of an electromagnetic field (i.e. in assumption that electromagnetic field include only longitudinal modes).

Based on Carson's work, Wise [10, 11] proposed more general expression considered the displacement currents in the air and the ground. Sunde [12] summarized these works and proposed generalized formula. Later some approximate expressions were proposed by Gary [13], Deri et al [14], F. Rachidi [15] and others.

It has been shown in [15] that the validity of the Carson's approximation extends to frequencies of about a few MHz for typical overhead power lines and for ground conductivities of about 0.01 Sm/m.

Theoretical consideration of the frequency dependence traction rails impedance is more complicated task due to

- complex structure of the railroad track consisting of two rails, sleepers, ballast;
- skin-effect in rails;
- strong current dependence of magnetic permeability of rail steel;
- influence of the nearly lied ground due to displacement current being inducted in the ground and leakage current due to small resistance between rail and a ground;
- complex shape of rail cross-cut.

Available data of rail impedance investigations [4 - 7] are related mainly to the 1435 mm gauge rail system and UIC 60 rail type. Reference data for electric impedance of traction rails R65 type and 1520 mm gauge are given in table 1 [17]. Plot of frequency dependence of impedance built according to [17] has the broken polygonal form, which is possible due to the measurement error. Results of rail impedance calculations for the traction current harmonics according to Carson's method didn't provide satisfactory concordance with the data [17] for frequencies greater than 1 kHz.

Table 1

Traction rail impedance [17]

f, Hz	Z, Ohm/km	f, Hz	Z, Ohm/km
25	0.308+0.394i	580	1.077+6.106i
50	0.338+0.725i	720	1.221+7.299i
75	0.401+0.992i	780	1.236+7.803i
175	0.618+1.902i	4545	1.529+43.773i
420	0.935+4.810i	5000	1.700+48.670i
480	0.938+5.318i	5555	1.871+53.567i

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However for the many practical important tasks it is necessary to know the frequency dependence of rail impedance for traction rails R65 type and 1520 mm gauge at all frequencies in audiofrequency range ($10^0 \ldots 10^4$ Hz).

The aim of the work is analysis and study of the validity of the basic methods proposed for determination of a serial impedance of the conductors above a lossy ground, conformably to the impedance calculation of R65 type traction rails with 1520 mm gauge in the tonal frequencies range. To achieve this goal a brief mathematical formulation of the impedance calculation methods for conductors above lossy ground were carried out. According to Carson's method and complex depth of earth return method the serial impedance of the traction rails 1520 mm gage were calculated and results were compared with literature data.

Mathematical Formulation

Analysis of the electrical processes in the rail circuits is usually carried out on the basis of the multiconductor transmission line theory with the representation of the lines longitudinal impedance and transverse admittance in a matrix form. For track consisting of two rails above lossy ground the diagonal elements of a series impedance matrix Z_{ii} $(i = 1,2)$ are the values of a rail self impedances (p.u.l.), defined as the ratio of the voltage drop (p.u.l.) to the current flowing in the rail and returning through the earth.

Off-diagonal elements of a series impedance matrix Z_{ij} $(i, j = 1,2)$ are mutual impedances between i-th and j-th conductors and defined as the ratio of the induced voltage (p.u.l.) in i-th conductor to the current in j-th conductor. Both the self and mutual impedances are affected by the earth return current.

Self impedance

Self impedance of the conductor is a sum of three components as follows

$$Z_{ii} = Z_{Cii} + Z_{Eii} \tag{1}$$

where Z_{Cii} is the internal impedance of the line conductor, Z_{Eii} - is the external impedance of the conductor equal to the sum of the Z_{Egii} - geometric impedance due to reactance involved in the magnetic field in the air (external inductance), and Z_{gii} is the ground-return impedance of the conductor (due to the earth contribution).

$$Z_{Eii} = Z_{Egii} + Z_{gii} \ . \tag{2}$$

The external self-impedance of the conductor is equal $Z_{Eii} = j\omega L_{ii}$, in which L_{ii} is an external inductance of the conductor that is given by

$$L_{ii} = \frac{\mu_0}{2\pi} \ln \frac{2h_i}{r_i} \tag{3}$$

where $\mu_0 = 4\pi\ 10^{-7}$ H/m is the magnetic permeability constant, h_i is the height of the conductor above the ground and r_i is the radius of the conductor.

The ground-return impedance of the conductor can be written as

$$Z_{gii} = R_{gii} + X_{gii}. \tag{4}$$

The mutual impedance

The mutual impedance Z_{ij} of two conductors i and j can be written as

$$Z_{ij} = Z_{mij} + Z_{gij}, \tag{5}$$

where $Z_{mij} = j\omega L_{ij}$ - is the impedance due to mutual inductance L_{ij} between the two conductors supposing the conductors and the ground are perfectly conductive and Z_{gij} is the impedance of the earth return path that is common to the currents in conductors i and j. The mutual inductance L_{ij} is defined by geometric parameters of the line conductor system (fig. 1) and can be written as

$$L_{ij} = \frac{\mu_0}{2\pi} \ln \frac{D_{ij}'}{D_{ij}} = \frac{\mu_0}{2\pi} \ln \sqrt{\frac{d_{ij}^{\ 2} + \left(h_i + h_j\right)^2}{d_{ij}^{\ 2} + \left(h_i - h_j\right)^2}} \tag{6}$$

where D_{ij} is the distance between conductors i and j, and D_{ij}' is the distance between conductor i and the image of conductor j; other geometric parameters are illustrated in fig. 1.

The ground-return impedance (due to the earth contribution) can be written as

$$Z_{Gij} = R_{Gij} + jX_{Gij}. \tag{7}$$

The internal impedance of a conductor is given by

$$Z_{Cii} = R_{Cii} + jX_{Cii} \tag{8}$$

where R_{Cii} is the internal resistance and $X_{Cij} = \omega L_{Cii}$ is the internal reactance relevant to the internal inductance L_{Cii}.

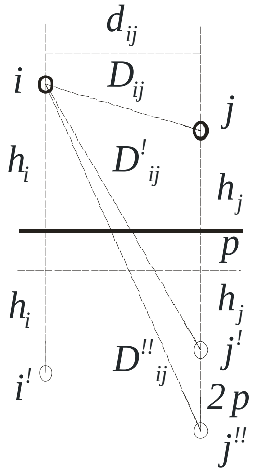


Fig. 1. Geometry of conductors i, j and their images

Assuming uniform current density the internal self resistance and inductance of a circular cross-section conductor are given by well-known simple formulae available for the preliminary calculation

$$R_{Cii} = (\sigma S)^{-1}, \quad L_{Cii} = \frac{\mu\mu_0 r_i}{8\pi} \tag{9}$$

where σ is the conductivity of a conductor material, S is a cross-sectional area of the conductor, μ - relative magnetic permeability.

The radius of an equivalent circular cross-section conductor for a rail may be evaluated with two different expressions [7, 16]. At low frequency, where the current is distributed almost uniformly across the rail section, equivalent radius may be computed as

$$r_i = \sqrt{S/\pi}. \tag{10}$$

At high frequency, where the current is distributed almost along the conductor perimeter, equivalent radius may be computed as

$$r_i = P_r/2\pi, \tag{11}$$

where P_r is a cross-section rail perimeter.

Expressions (9)- (11) doesn't provide accurate estimation of rail internal self resistance and inductance due to a complex rail cross-section shape, a skin effect at AC and changing of a magnetic permeability with current variation at DC. For more precision estimation of a rail internal self-impedance a finite-element method (FEM) was used [0].

In present work for the calculation of a rail internal impedance was used approximated method based on equivalent cylindrical conductor model considering a skin effect [0]. According to this method the internal resistance of a cylindrical conductor is given by

$$R_{Cii} = \frac{R_S}{\sqrt{2\pi r_i}} \frac{Ber(q)Bei'(q) - Bei(q)Ber'(q)}{\left(Ber'(q)\right)^2 + \left(Bei'(q)\right)^2},$$

$$X_{Cii} = \frac{R_S}{\sqrt{2\pi r_i}} \frac{Ber(q)Ber'(q) - Bei(q)Bei'(q)}{\left(Ber'(q)\right)^2 + \left(Bei'(q)\right)^2},$$

$$q = \sqrt{2r_i/\delta} \tag{12}$$

where $R_S = (\sigma\delta)^{-1}$ is a surface resistance, $\delta = \left(\sqrt{\pi f \mu\sigma}\right)^{-1}$ is a depth of penetration, *ber, bei, ker* and *kei* are Kelvin's functions which belong to the Bessel function family, and *ber', bei', ker'* and *kei'* are their derivatives, respectively. Kelvin's functions are often defined as

$$Ber(q) + jBei(q) = J_0\left(j^{-1/2}\right) \tag{13}$$

where J_0 is the modified Bessel functions of the 1st kinds of order zero.

The correction terms for the self-impedance of i-th conductor and mutual impedance of two conductors i and j due to earth path impedance are derived by Carson [0]:

$$Z_{Gii} = \frac{j\omega\mu}{\pi} \int_0^\infty \frac{\exp(-2h_i\xi)}{\xi + \sqrt{\xi^2 + j\omega\mu\sigma}} d\xi, \tag{14}$$

$$Z_{Gij} = \frac{j\omega\mu}{\pi} \int_0^\infty \frac{\exp\left[-\left(h_i + h_j\right)\xi\right]}{\xi + \sqrt{\xi^2 + j\omega\mu\sigma}} d\xi \ . \tag{15}$$

More general expression for mutual ground impedance between two conductors i and j applicable to higher frequency band derived by Sunde [0] is given by

$$Z_{Gij} = \frac{j\omega\mu}{\pi} \int_0^\infty \frac{\exp[-(h_i + h_j)\xi]}{\xi + \sqrt{\xi^2 + \gamma_g^2}} \cos(d_{ij}\xi) d\xi \qquad (16)$$

where γ_g is a wave propagation constant defined as

$$\gamma_g = \sqrt{j\omega\mu_0(\sigma_g + j\omega\varepsilon_0\varepsilon_g)}. \qquad (17)$$

These expressions contain infinite integrals with complex arguments. For theirs evaluation Carson has proposed infinite series

$$R_{Gii} = 4\omega 10^{-7}\left\{\frac{\pi}{8} - b_1 k + b_2(C_2 - \ln k)k^2 + \right. \\ \left. + b_3 k^3 - d_4 k^4 - ...\right\}, \qquad (18)$$

$$X_{Gii} = 4\omega 10^{-7}\left\{\frac{1}{2}(0.6159315 - ln\,k) + b_1 k - \right. \\ \left. - d_2 k^2 + b_3 k^3 - b_4(C_4 - ln\,k)k^4 + ... \right\}, \qquad (19)$$

$$R_{Gij} = 4\omega 10^{-7}\left\{\frac{\pi}{8} - b_1 k_m \cos\theta + b_2[(C_2 - \right. \\ - \ln k_m)k_m{}^2 \cos 2\theta + \theta k_m^2 \sin 2\theta] + \\ \left. + b_3 k_m{}^3 \cos 3\theta - d_4 k_m{}^4 \cos 4\theta - ...\right\}, \qquad (20)$$

$$X_{Gij} = 4\omega 10^{-7}\left\{\frac{1}{2}(0.6159315 - ln\,k_m) + \right. \\ + b_1 k_m \cos\theta - d^2 k_m{}^2 \cos 2\theta + b^3 k_m{}^3 \cos 3\theta - \\ \left. - b_4[(C_4 - ln\,k_m)k_m^4 \cos 4\theta + \theta k_m^4 \sin 4\theta]..\right\}. \qquad (21)$$

where

$$b_1 = \sqrt{2}/6\,;\;\; b_2 = 1/16\,;\;\; b_i = b_{i-2}\frac{sign}{i(i+2)}; \qquad (22)$$

$$C_1 = 1.3659315;\;\; C_i = C_{i-2} + \frac{1}{i} + \frac{1}{i+2}; \qquad (23)$$

$$d_i = \frac{\pi}{4} b_i; \quad \theta = \arcsin\left(\frac{d_{ij}}{D_{ij}}\right); \qquad (24)$$

$$k = 4\pi\sqrt{5}\ 10^{-4} 2h_i \sqrt{f\sigma_g}; \qquad (25)$$

$$k_m = 4\pi\sqrt{5}\ 10^{-4} D_{ij}' \sqrt{f\sigma_g}\ . \qquad (26)$$

These approximations are valid for a limited range of frequencies, and medium frequencies are not covered [16].

The Complex Depth of Earth Return method

The complex depth of earth return method [13, 14] assumes that the current in conductor i returns through an imagined earth path located directly under the original conductor at a depth of $(h_i + 2p)$ as shown in fig. 1, where p *is* the skin depth of the ground. Thus, the self and the mutual impedances can be written as

$$Z_{Eii} = j\omega\frac{\mu_0}{2\pi}\ln\frac{2(h_i + p)}{r_i}; \qquad (27)$$

$$Z_{ij} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D_{ij}'}{D_{ij}} = \\ = j\omega\frac{\mu_0}{2\pi}\ln\sqrt{\frac{d_{ij}^2 + (h_i + h_j + 2p)^2}{d_{ij}^2 + (h_i - h_j)^2}} \qquad (28)$$

where $p = \sqrt{\rho / j\omega\mu_0}$.

Results

The impedance of traction rails R65 type and 1520 mm gauge were calculated using formulas (1)-(12). The correction terms for the self- and mutual impedance of rails due to earth path impedance were determined using Carson's method with representation of integrals as infinite series (14)-(26) and complex depth of earth return method (27)-(28). Calculations have been performed for such parameters of traction rail system: perimeter of rail cross-section P_r =0,7 m, distance between rails' axes $d_{ij} = 1.6\ m$, height rails above a ground $h_i = 0.5\ m$, conductivity of the ground $\sigma_g = 0.1\ Sm/m$, steel resistivity $\sigma_S = 0.21\ Ohm\ mm^2/m$, steel relative permeability μ =100.

The rail loop electrical parameters (p.u.l.) are obtained as

$$R = (R_1 + R_2) = 2R_1;$$

$$L = 2(L_1 - L_{12})$$

where $R_1 = R_2$ and $L_1 = L_2$ - self-resistance and self inductance of rails, respectively, L_{12} - mutual inductance of rails.

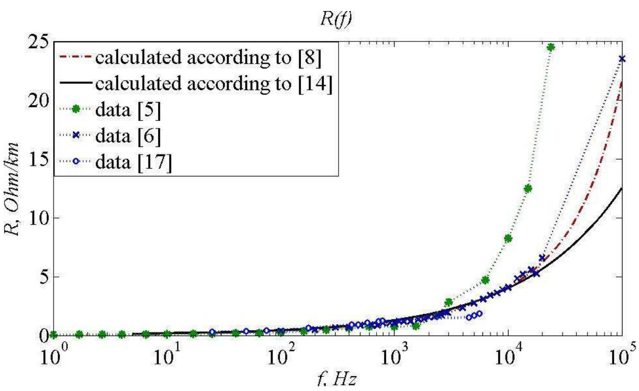
Frequency dependences of the calculated active resistance R and inductance L (p.u.l.) of rail loop for tracks 1520 mm width and P65 rail type are shown in fig. 2.

Also in fig. 2 the frequency dependences of the rail loop resistance and inductance (p.u.l.) for tracks 1520 mm plotted according to reference data [17] are shown. Plot according to data [17] looked like zigzag broken line that probably due to precision of measurements.

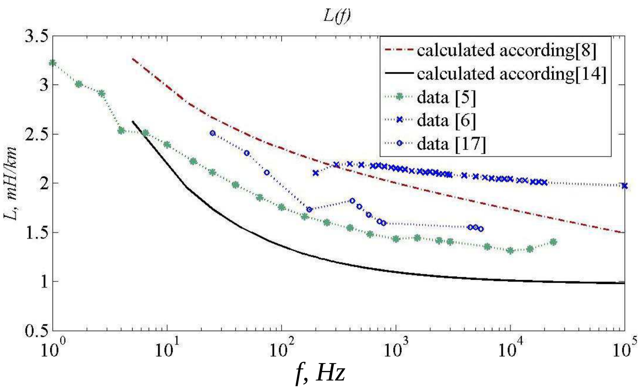
Values of rail loop resistance R calculated by Carson's method and complex depth of earth return method are in good agreement with the data [17] (fig. 2), while the values of inductance L calculated by Carson's method and the complex depth of earth return method are quite different to each other and to data [17].

The resistance and inductance of the traction rail of 1435 gauge [4 - 7] are also represented in fig. 2 for comparison. Date [4, 6] were measured for railroad section with length of 36 m, type of sleepers - concrete and wooden, frequency range – 1 Hz..25 kHz.

Serial rail loop resistance and inductance were measured in [6] for loop circuit with the UNI 60 type of rails, cross-section area – 7679 mm2, full length - 5.8 m, length between voltage terminals - 5.2 m. The date of [6] were calculated on 1 km length and also shown in fig. 2.



(a)



(b)

Fig. 2. Frequency dependences of the resistance R (a) and inductance L (b) (p.u.l.) of for traction rails loop

Since data [4 - 7] have been obtained for the other types of tracks with 1435 mm gauge, and other types of rails, sleepers etc., these data were different from data for traction rails of 1520 mm gauge and presented in fig. 2 for qualitative comparison.

In this way, it may be concluded that the behavior of the frequency dependences of traction rail's impedance obtained by calculations according to Carson's method and complex depth of earth return method are in good qualitative agreement with the data [4 - 7] in frequency range $10^0..10^5$ Hz.

The numerical values of the results calculated for tracks 1520 mm type according to Carson's method and complex depth of earth return method differ from reference data [17], and these differences were increased with increasing of frequency.

These differences may be explained by differences of numerical values of the basic electrical parameters of the traction systems using for calculations in present work and parameters of the systems under measuring [0-0]. Another reasons of obtained differences in calculated and measured results may be due to error of calculation methods [8, 9, 13, 14] caused by small height ($h_i \leq 1\ m$) of rails above lossy ground and due to electrical connection between rails and a ground.

Such miscalculation can be eliminated by using correction factors in calculation expressions (fig. 2).

Conclusion

Based on the comparative analysis of the results of calculations of the series impedance of R65 type traction rails with 1520 mm gauge in the audiofrequency range according to Carson's method and complex depth of earth return method and literature data it have been confirmed applicability of the using of these methods for calculations of rail's impedance for 1520 mm gauge in audiofrequency range with using of correction factors.

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Гаврилюк В.И., **Мелешко В.В. Электрический импеданс тяговых рельсов в диапазоне тональных частот.** На основе сравнительного анализа результатов вычислений последовательного импеданса рельсов типа Р65 стандарта 1520 мм в диапазоне тональных частот методом Карсона и методом комплексной толщины поверхностного слоя земли, а также литературных данных, подтверждена применимость рассмотренных методов для вычисления импеданса рельсов стандарта 1520 мм в диапазоне тональных частот при условии использования корректирующих коэффициентов.

Ключевые слова: рельсы, частотно-зависимый импеданс, метод Карсона, метод комплексной толщины поверхностного слоя земли.

Гаврилюк В.І., **Мелешко В.В. Електричний імпеданс тягових рейок в діапазоні тональних частот**. На основі порівняльного аналізу результатів розрахунку послідовного імпедансу рейок типу Р65 стандарту 1520 мм в діапазоні тональних частот методом Карсона и методом комплексної товщини поверхневого шару землі та літературних даних, підтверджено можливість використання розглянутих методів для обчислення імпедансу рейок стандарту 1520 мм в діапазоні тональних частот при умові використання коректуючи коефіцієнтів.

Ключові слова: рейки, частотно-залежний імпеданс, метод Карсона, метод комплексної товщини поверхневого шару землі.

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